CE 418.3 - Design in Reinforced Concrete

MIDTERM EXAMINATION

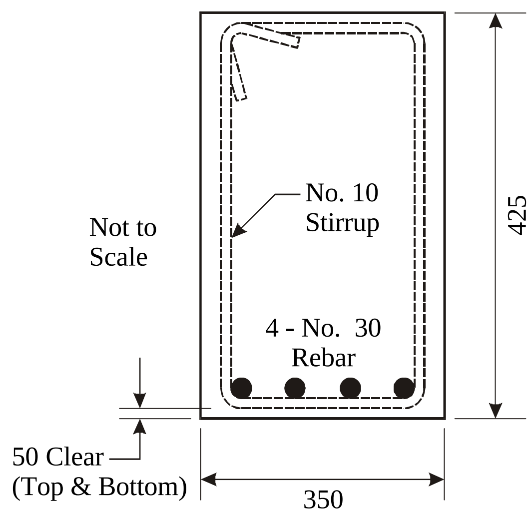
October 30, 2003

Time Allowed: 2 Hours | Professor: B. Sparling

Notes:
- Closed book examination
- CPCA Concrete Design Handbook may be used
- Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (i.e. formulas)
- Show **all** your work, including all formulas and calculations

MARKS

QUESTION 1: The reinforced concrete beam shown on the right is constructed with Grade 400 reinforcing steel and concrete with a 28 day design strength of $f_c' = 25$ MPa.

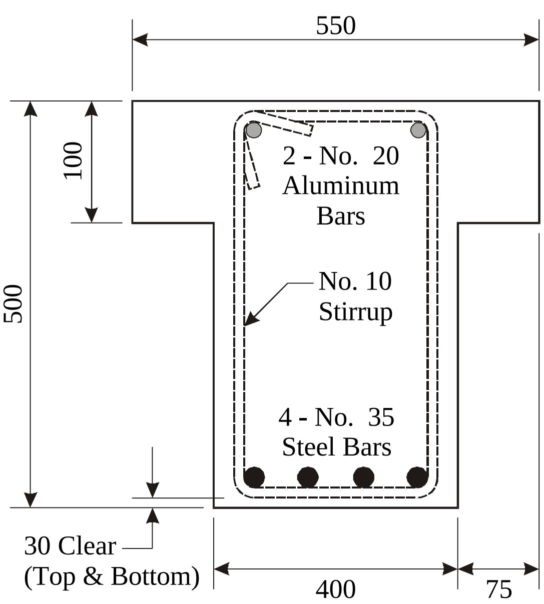


20 **(a)** Calculate the ultimate positive bending moment resistance M_r in accordance with the requirements of CSA-A23.3-94 Clauses 10.1 and 10.1.7. Assume that the tension steel does not yield and check that assumption.

10 **(b)** Determine the area of compression reinforcing steel A_s' that would be required to produce a **balanced** failure condition in the beam. Do not select the actual bars. Assume that the compression steel yields, and check this assumption.

26 **QUESTION 2:** The reinforced concrete T-beam shown below was tested in the Structures Laboratory under loading conditions that produced positive bending moments throughout. The beam was observed to exhibit a flexural failure at midspan.

The beam was constructed with concrete having a compressive strength of $f_c' = 30$ MPa. In addition to the **4 – No. 35 Grade 425 steel rebar** on the bottom face of the beam, the beam featured **2 – No. 20 aluminum bars**, each with a nominal cross-sectional area of **310 mm^2**, used as compression reinforcement.



The aluminum bars exhibit a nonlinear stress-strain relationship defined by the expression:

$$f_a = 69\times10^3\ e_a - 8.62\times10^6\ e_a^2$$

[MPa]

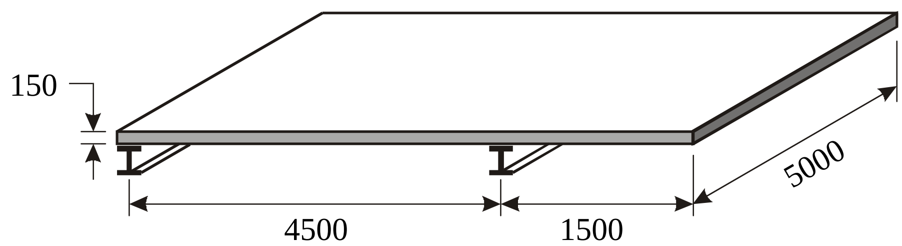
where f_a is the stress in the aluminum [in MPa] at a strain of e_a.

At the instant of failure, the measured value of the strain in the aluminum rebar at midspan was found to be $\varepsilon_a = 0.00232$.

Estimate the **nominal moment capacity**, M_n, of the beam at failure. Assume the concrete behaviour at failure can be adequately described by CSA A23.3-94 Clause 10.1, including the equivalent rectangular concrete stress distribution described in Clause 10.1.7 (without the material resistance factor ϕ_c).

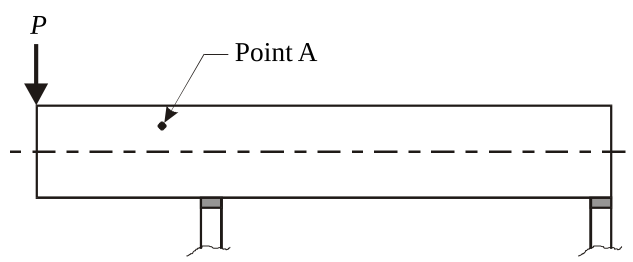
26 **QUESTION 3:** A 150 mm thick concrete slab spans one-way between steel support beams, as shown below, and cantilevers out an additional 1.5 m beyond the right-most support beam. The slab supports a uniformly distributed superimposed dead load of $q_D = 1.25\ \text{kPa}$ and a live load of $q_L = 4.8\ \text{kPa}$. The slab is constructed with Grade 400 reinforcing steel and concrete with a 28 day design strength of $f_c' = 30\ \text{MPa}$.

Using the "unit design strip" approach and Table 2.1 in Part II of the CPCA Concrete Design Handbook, select the principal reinforcement for the negative moment region above the right-most support beam in accordance with provisions of CSA-A23.3-94. Assume a clear concrete cover of 20 mm on the principal reinforcement. Verify that Table 2.1 is applicable for this case.



QUESTION 4: Provide **brief** answers to the following questions. Answers in point form are acceptable. Sketches should be used to supplement written responses if appropriate.

7 **(a)** Predict the approximate orientation of the crack that would form at Point A in the reinforced concrete beam below. Justify your answer.



6 **(b)** List three of the primary mechanisms contributing to the shear resistance of concrete beams without stirrups.

5 **(c)** Describe the effect that the addition of compression reinforcement A_s' will have on the strain in the principal tensile reinforcement A_s. Justify your answer.

<u>Supplemental Material:</u>

- **Material Properties:** $\phi_c = 0.6$ $\phi_s = 0.85$ $\alpha_D = 1.25$ $\alpha_L = 1.5$

$$f'_{ct} = \frac{t}{\alpha + \beta t}\ f'_c \qquad \frac{f_c}{f'_c} = 2\left(\frac{\varepsilon_c}{\varepsilon'_c}\right) - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2 \qquad f_{ct} = \frac{2P}{\pi d L} \approx 0.53\sqrt{f'_c}$$

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\gamma_c/2300\right)^{1.5} \qquad E_s = 200{,}000\ \text{MPa} \qquad \varepsilon_{cu} = 0.0035$$

$$f_r = 0.6\,\lambda\sqrt{f'_c} \qquad \gamma_c = 2400\ \text{kg/m}^3$$

- **Flexural Analysis:** $\Sigma F_x = 0$ $\Sigma M = 0 \rightarrow M = T\,(j\,d) = C_c\,(j\,d)$

$$C_c = \int_0^c f_c\,\mathrm{d}A \qquad \bar{y}\,C_c = \int_0^c y\,f_c\,\mathrm{d}A \qquad C_c = \left(\phi_c\ \alpha_1\ f'_c\right)(\text{Area}) \qquad T = \phi_s\ A_s\ f_s$$

$$\alpha_1 = 0.85 - 0.0015\ f'_c \geq 0.67 \qquad \beta_1 = 0.97 - 0.0025\ f'_c \geq 0.67 \qquad a = \beta_1\,c$$

$$a = \frac{\phi_s\ A_s\ f_s}{\phi_c\ \alpha_1\ f'_c\,b} \qquad \varepsilon_s = \varepsilon_{cu}\left(\frac{d-c}{c}\right) \qquad \frac{c}{d} \leq \frac{700}{700 + f_y} \qquad \frac{d'}{c} \leq 1 - \frac{f_y}{700}$$

$$\left(A_s\right)_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1\,b\,d}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad A_{s1} = A'_s\left(\frac{f'_s}{f_s} - \frac{\phi_c\ \alpha_1\ f'_c}{\phi_s\ f_s}\right) \qquad A_{s2} = A_s - A_{s1}$$

$$M_{r1} = \phi_s\ A_{s1}\ f_{s1}\left(d - d'\right) \qquad M_{r2} = \phi_s\ A_{s2}\ f_{s2}\left(d - \frac{a}{2}\right) \qquad \varepsilon'_s = \varepsilon_{cu}\left(\frac{c - d'}{c}\right)$$

- **Flexural Design:** $A_{s_{\min}} = \dfrac{0.2\sqrt{f'_c}}{f_y}\ b_t\,h$ $\rho = \dfrac{A_s}{b\,d}$ $K_r = \dfrac{M_r \times 10^6}{b\,d^2}$

$$\rho_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad K_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right) \qquad M_r \geq M_f$$

$$M_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right)b\,d^2 \qquad \rho = \frac{\phi_c\ \alpha_1\ f'_c \pm \sqrt{\left(\phi_c\ \alpha_1\ f'\right)^2 - 2\,K_r\ \phi_c\ \alpha_1\ f'}}{\phi_s\ f_y}$$

- **One-Way Floor Systems:** $A_{s_{\min}} = 0.002\ A_g$ $A_{sh} = \dfrac{\left(\phi_c\ \alpha_1\ f'_c\right)\left(h_F\ b\right)}{\phi_s\ f_y}$